This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1155**

Unique Paper Code : **237253** **G**

Name of the Paper : **Algebra II, Paper STH-202**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all.

Question No. 1 is compulsory.

Attempt *five* more questions selecting at least *two* questions from each Section.

1. (*a*) State whether the following statements are true *or* false :

(*i*) Inverse of $E_{ij}(k) \neq 0$ is $E_{ij}\left(\frac{1}{k}\right)$.

(*ii*) A characteristic root of a skew-Hermitian matrix is either unity or a pure imaginary number.

(*iii*) Generalized inverse of a matrix of any order always exists.

(*iv*) The equation AX = 0 has a non-zero solution if and only if the rank '*r*' of A is less than the number '*n*' of its columns i.e. of the unknowns.

(*v*) N, the set of natural numbers is a group with respect to subtraction.

(*b*) Let A be any $m \times n$ matrix such that $\rho(A) = r\ (>0)$. What will be its normal form ?

(*c*) Is the matrix :

$$\begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

equivalent to I_3 ? Justify.

(*d*) Write down the elementary matrix $E_{32}(2)$ of order 3 × 3 and its inverse.

(*e*) Define space discriminant and signature of a quadratic form.

(*f*) If A is a $m \times n$ matrix of rank *p*, L and B are two non-singular matrices of order *m* and space *n* respectively, then write down the form of LAB. 5,2,2,2,2,2

Section I

2\. (*a*) Define the rank of a given matrix. Show that the rank of a matrix is invariant under the elementary row operation :

$$R_i \to R_i + \lambda R_j;\quad \lambda \neq 0.$$

(*b*) Investigate for what values of λ and μ the system of simultaneous equations :

$$\begin{aligned} x + 2y + 3z &= 5 \\ 3x - y + 2z &= 12 \\ 3x - y + \lambda z &= \mu \end{aligned}$$

have :

(*i*) no solution

(*ii*) a unique solution and

(*iii*) an infinite number of solutions. 6,6

3\. (*a*) State and prove "Cayley Hamilton" theorem.

(*b*) Find the characteristic roots of the matrix : 6,6

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

and show that the characteristic vectors associated with its distinct characteristic roots are mutually orthogonal.

4. (a) Identify the nature of the quadratic form :

$$4x^2 + 9y^2 + 2z^2 + 8yz + 6zx + 6xy$$

and hence find the rank, index and signature of the form.

(b) Prove that the modulus of each characteristic root of a unitary matrix is unity. 7,5

5. (a) Express :

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

as a product of space elementary matrices.

(b) Prove that the definiteness of a quadratic form is invariant under non-singular linear transformation. 7,5

Section II

6. (a) If G is a generalized inverse of X' X, then prove that :

(i) G' is also a generalized inverse of X' X.

(ii) XGX'X = X, ie. GX' is a generalized inverse of X.

(iii) XGX' is invariant to G

(iv) XGX' is symmetric whether G is symmetric or not.

(b) Compute the inverse of partitioned matrix :

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

of order $n \times n$, assuming A_{22} is a non-singular submatrix of order $s \times s$. 7,5

7. (a) Prove that the totality of all positive rational numbers forms an abelian group under the composition defined by $a*b = (ab)|2$.

(b) If $2Z = \{2m : m \in Z\}$, Z is a set of all integers. Show that (2Z, +, .) is a commutative ring without unity and without divisors of zero. How about 3Z ? How about kZ for $k \in N$, where N is a set of all natural numbers? 6,6

8. (a) Define orthonormal basis. Using the Gram Schmidt orthogonalisation space process construct an orthonormal basis for E^3 from the following set of basis vectors :

$a_1 = [1, 1, 1,]$; $a_2 = [0, 1, 1]$; $a_3 = [0, 0, 1]$

(b) (i) Do the vectors (1, 1, 0), (0, 1, 2) and (0, 0, 1) form a basis of $V_3(R)$?

(ii) Express the vector V = (3, 1, –4) as a linear combination of the vectors $V_1 = (1, 1, 1)$, $V_2 = (0, 1, 1)$ and V_3 (0, 0, 1). 6,3,3